

# Anomalous Band Structure in Odd-Odd Nuclei with the Quadrupole-Quadrupole Interaction

M.S. Fayache<sup>1</sup> and L. Zamick<sup>2</sup>

(1) Département de Physique, Faculté des Sciences de Tunis  
Tunis 1060, Tunisia

(2) Department of Physics and Astronomy, Rutgers University  
Piscataway, New Jersey 08855

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## Abstract

We perform shell model calculations in odd-odd nuclei using a quadrupole-quadrupole interaction with single-particle splittings chosen so as to obtain the  $SU(3)$  results. Elliott had shown that such an interaction gives rotational bands for which the energies go as  $I(I+1)$ . This certainly is true for even-even and for odd-even or even-odd nuclei with  $K \neq 1/2$ . We have looked at odd-odd nuclei e.g.  $^{22}\text{Na}$  and found somewhat different behaviour. In  $^{22}\text{Na}$  the  $I = 1_1^+ T = 0$  and  $I = 0_1^+ T = 1$  states are degenerate, and a rotational band built on the  $I = 0_1^+ T = 1$  state behaves in a normal fashion. For the  $I = 1_1^+ T = 0$  band however, we find that the energy is given by  $E(I) - E(1_1^+) = AI(I+1)$ . This differs from the ‘normal’ behaviour which would be  $E(I) - E(1_1^+) = AI(I+1) - 2A$ .

## I. INTRODUCTION

In the rotational model the formula for the energy of a state in a rotational band with total angular momentum  $I$  is given by [1]

$$E_I = E_0 + \frac{\hbar^2}{2\mathcal{J}} \left[ I(I+1) + \delta_{K,1/2} a (-1)^{I+1/2} (I+1/2) \right] \quad (1)$$

where  $a$  is the decoupling parameter given by  $a = -\langle K = 1/2 | J_+ | \overline{K = 1/2} \rangle$  and where if  $|K\rangle = \sum_j C_{j,k} \phi_{j,k}$  then  $|\bar{K}\rangle = \sum_j C_{j,k} (-1)^{j+k} \phi_{j,-k}$ .

For even-even nuclei, and for odd-even and even-odd nuclei with  $K \neq 1/2$ , one gets the familiar  $I(I+1)$  spectrum [2].

It is generally thought that the Elliott  $SU(3)$  model also gives an  $I(I+1)$  spectrum. This has been discussed most explicitly in the context of even-even nuclei. The  $SU(3)$  results also give the more complex  $K = 1/2$  behaviour where the decoupling parameter  $a$  has a value corresponding to that obtained from an asymptotic Nilsson wave function. This will be discussed briefly in section II. But the main thrust of this work will be to show that for odd-odd nuclei one obtains in certain cases deviations from the above formula.

We have performed shell model calculations with all possible configurations in a given major shell using the interaction  $\sum_{i<j} Q(i) \cdot Q(j)$  where, in order to get Elliott's  $SU(3)$  results we must also add single-particle splittings, e.g. in the  $1s - 0d$  shell we have  $\epsilon_{0d} - \epsilon_{1s} = 18\bar{\chi}$  and in the  $1p - 0f$  shell we have  $\epsilon_{0f} - \epsilon_{1p} = 30\bar{\chi}$ , where  $\bar{\chi} = \frac{5b^4\chi}{32\pi}$  with  $b$  the oscillator length parameter ( $b^2 = \frac{\hbar}{m\omega}$ ).

As has been previously noted [3,4], we use the  $\vec{r}$ -space  $Q \cdot Q$  interaction rather than the mixed  $\vec{r}$  and  $\vec{p}$ -space one. With such an interaction 2/3 of the above single-particle splitting comes from the  $i = j$  part of  $Q \cdot Q$  and 1/3 from the interaction of the valence particle with the core.

## II. A BRIEF LOOK AT $K = 1/2$ BANDS

Let us be specific and discuss  $^{19}F$  and  $^{43}Sc$ . We consider in each case three valence nucleons beyond a closed shell. In  $^{19}F$  the particles are in the  $1s - 0d$  shell, whereas in  $^{43}Sc$  they are in the  $1p - 0f$  shell. The energy levels of the lowest bands are given in Table I for the two cases. The results for the two nuclei are striking but different. In  $^{19}F$ , the lowest state is a  $I = 1/2^+$  singlet, and at higher energies we get degenerate pairs  $(3/2^+, 5/2^+)$ ,  $(7/2^+, 9/2^+)$ ,  $(11/2^+, 13/2^+)$ . In  $^{43}Sc$  the ground state is degenerate, and the degenerate pairs are  $(1/2^+, 3/2^+)$ ,  $(5/2^+, 7/2^+)$ , ...,  $(17/2^+, 19/2^+)$ .

If we look at the rotational formula, we find that these results are consistent with a decoupling parameter  $a = +1$  for  $^{19}F$  and  $a = -1$  for  $^{43}Sc$ . It is easy to show that these are precisely the results one obtains with asymptotic Nilsson wave functions. In both cases the odd particle will be in a  $\Lambda = 0 \Sigma = 1/2$  state in the asymptotic limit. From the definition of  $\bar{K}$ , the state  $|\Lambda = 0 \bar{\Sigma} = 1/2\rangle$  can be shown to be equal to  $-(-1)^\pi |\Lambda = 0 \Sigma = -1/2\rangle$  where  $\pi$  is  $(+)$  for an even-parity major shell and  $(-)$  for an odd-parity one. Hence:

$$a = (-1)^\pi \langle \Sigma = +1/2 | J_+ | \Sigma = -1/2 \rangle = (-1)^\pi$$

It has long ago been noted by Bohr and Mottelson [1] that  $a = +1$  corresponds to weak coupling of the odd particle to  $I = 0^+, 2^+, 4^+, \dots$  states, whereas  $a = -1$  corresponds to weak coupling to  $I = 1, 3, 5, \dots$  states. It should be emphasized that the results in Table I are not the realistic ones -they represent the asymptotic extremes.

At any rate, we have shown that the  $Q \cdot Q$  interaction gives the same results for these two  $K = 1/2$  bands as does the rotational formula with asymptotic Nilsson wave functions.

### III. ODD-ODD NUCLEI E.G. $^{22}\text{Na}$

#### A. The Energy Spectra

In table II we show a fairly detailed list of energy levels for the odd-odd nucleus  $^{22}\text{Na}$  obtained with the  $Q \cdot Q$  interaction. We show  $T = 0$  and  $T = 1$  states in separate columns. We have underlined  $T = 0$  and  $T = 1$  rotational bands, and will now discuss them in more detail. We use the same parameters as in  $^{19}\text{F}$  just to bring out some similarities. If one is interested in a best fit, one should of course have an  $A$  dependence in  $\chi$ .

Note that the ground state consists of two degenerate states, one with  $I = 1^+ T = 0$  and the other with  $I = 0^+ T = 1$ . Both states have  $L = 0$  and the simple spin-independent interaction gives the same energy for  $S = 0$  and  $S = 1$ . Let us first look at the  $T = 1$  states. The ground state is  $I = 0^+$ . The  $2^+$  state is at 1.588 and is doubly degenerate. If we follow the rotational sequence  $I = 0^+, 2^+, 4^+, \dots$  we see a simple rotational behaviour:

$$E(I) - E(0_1^+) = AI(I + 1) \quad \left[ A = \frac{\hbar^2}{2\mathcal{J}} = E(2^+)/6 \right]$$

There is nothing new here.

We next look at the  $T = 0$  states. The lowest state has  $I = 1^+$  (it is degenerate with the lowest  $I = 0^+ T = 1$  state). If we follow the underlined states we have a  $2^+$  at 1.588 MeV,  $3^+$  at 3.177 MeV,  $4^+$  at 5.293 MeV,  $5^+$  at 7.941 MeV until we reach  $10^+$  at 29.117 MeV.

The energy levels of  $I = 2^+, 3^+, \dots, 9^+, 10^+$  are given by

$$E^*(I) \equiv E(I) - E(1_1^+) = AI(I + 1) \quad \left[ A = \frac{\hbar^2}{2\mathcal{J}} = E(2^+)/6 \right]$$

At first sight there would appear to be nothing wrong. But remember that  $E^*(I)$  is the energy of a state of angular momentum  $I$  for which the  $I = 1^+$  state has been set to zero energy. If we put  $I = 1$  into the above formula we would get  $E^*(1) = 2A$ .

To put it in a better way, the rotational formula at the beginning of this paper (Eq. (1)) would yield

$$E(I) - E(1^+) = AI(I + 1) - 2A$$

However, the results that we obtain are

$$\begin{aligned} E^*(I) = E(I) - E(1^+) &= AI(I + 1) & I \neq 1 \\ &= 0 & I = 1 \end{aligned}$$

Thus, for the case of  $T = 0$  states in odd-odd nuclei we get a difference between the rotational formula and the  $SU(3)$  limit.

### B. The $B(E2)$ Values for $T = 0 \rightarrow T = 0$ Transitions

To clarify the structure of these bands, we performed calculations of  $B(E2)$  values for various  $T = 0 \rightarrow T = 0$  transitions up to  $I = 4$ . They are shown in Tables III and IV, where we introduced a small spin-orbit splitting in order to remove the degeneracies as our shell model code does not handle transitions involving degenerate states very well. Note that with bare  $E2$  charges  $e_p = 1$ ,  $e_n = 0$  we obtain  $B(E2 : 1_1^+ T = 0 \rightarrow 2_1^+ T = 0) = 34.9 e^2 fm^4$ . This is quite large, and in our opinion justifies treating the  $I = 1^+$  state as a member of the band. Actually, if we used the usual effective charges  $e_p = 1.5$ ,  $e_n = 0.5$ , the  $B(E2)$  value would increase four-fold (i.e. to about  $140 e^2 fm^4$ ). Note also that the cross-over transition  $I = 1_1^+ T = 0 \rightarrow I = 3_2^+ T = 0$  at  $3.2 MeV$  is zero. This is consistent with the  $I_1^+$  state being  $L = 0 S = 1$  and the  $I = 3_2^+$  state being  $L = 3 S = 1$ . One cannot connect from  $L = 0$  to  $L = 3$  via the  $E2$  operator. There is some strength to a lower  $3^+$  state which is *not* a member of the rotational band ( $B(E2) = 6.15 e^2 fm^4$ ). That  $3^+$  state *must* be  $L = 2 S = 1$ .

Our work suggests that the rotational model formula requires an additional term for odd-odd nuclei in order to be consistent with the  $SU(3)$  results [2]. We gain further insight by examining the degeneracies associated with the  $T = 0$  underlined states of Table II, i.e. those with energy  $AI(I + 1)$ . The even  $I$  states up to  $I = 8$  are doubly degenerate whereas the others are singlets. This suggests that there are two bands for which the states with the same  $I$  values are degenerate. One band is a  $K = 2$  band with all values of  $I$  from 2 to 10, and there is nothing anomalous about it. The other band consists of states of angular momentum 1,2,4,6 and 8. For the latter band, the *orbital* angular momentum of the states are 0,2,4,6 and 8 respectively, and they all have  $S = 1$ . Their energies can be fit to the formula  $E^*(I) = BL(L + 1)$  rather than  $AI(I + 1)$ .

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# TABLES

TABLE I. Energy Levels (in  $MeV$ ) of Excited States Corresponding to the  $K = 1/2$  Ground State Bands in  $^{19}F$  and  $^{43}Sc$  with the  $-\chi Q \cdot Q$  Interaction.

$^{19}F^a$		$^{43}Sc^b$	
$I^\pi$	$E^*$	$I^\pi$	$E^*$
$(\frac{1}{2})^+$	0	$(\frac{1}{2})^-$	0
$(\frac{3}{2})^+$	1.588	$(\frac{3}{2})^-$	0
$(\frac{5}{2})^+$	1.588	$(\frac{5}{2})^-$	0.679
$(\frac{7}{2})^+$	5.295	$(\frac{7}{2})^-$	0.679
$(\frac{9}{2})^+$	5.295	$(\frac{9}{2})^-$	1.900
$(\frac{11}{2})^+$	11.118	$(\frac{11}{2})^-$	1.900
$(\frac{13}{2})^+$	11.118	$(\frac{13}{2})^-$	3.664
		$(\frac{15}{2})^-$	3.664
		$(\frac{17}{2})^-$	5.971
		$(\frac{19}{2})^-$	5.971

<sup>a</sup>For  $^{19}F$  we use  $\chi = 0.1841$  ( $\bar{\chi} = 0.0882$ )

<sup>b</sup>For  $^{43}Sc$  we use  $\chi = 0.0294$  ( $\bar{\chi} = 0.0218$ )

TABLE II. The Energy Levels (in  $MeV$ ) of  $^{22}Na$  Calculated with the  $-\chi Q \cdot Q$  Interaction<sup>a</sup>

$I^\pi$	$T = 0$ States	$T = 1$ States
$0^+$	8.999	<u>0.000</u>
	12.176	2.647
	12.176	8.999
	13.235	9.000
	16.410	12.176
$1^+$	<u>0.000</u>	2.647
	1.588	8.999
	1.588	8.999
	2.647	10.059
	9.000	10.059
$2^+$	<u>1.588</u>	<u>1.588</u>
	1.588	1.588
	3.176	2.647
	8.999	5.294
	10.059	9.000
$3^+$	1.588	3.176
	1.588	5.293
	<u>3.177</u>	10.058
	5.294	10.058
	5.294	11.646
$4^+$	3.176	<u>5.293</u>
	<u>5.293</u>	5.293
	5.293	5.293
	7.941	10.059
	11.647	11.647
$5^+$	5.294	7.941
	5.294	10.059
	<u>7.941</u>	13.763



	10.059	13.763
	11.118	13.763
6 <sup>+</sup>	7.941	10.059
	<u>11.117</u>	<u>11.117</u>
	11.117	11.118
	14.824	16.412
	16.411	16.412
7 <sup>+</sup>	11.117	<u>14.823</u>
	11.117	16.940
	<u>14.823</u>	19.587
	16.941	19.587
	19.058	19.587
8 <sup>+</sup>	14.823	16.941
	<u>19.058</u>	<u>19.058</u>
	19.059	19.059
	22.763	22.767
	23.292	23.293
9 <sup>+</sup>	19.058	<u>23.822</u>
	19.058	25.939
	<u>23.822</u>	26.470
	25.940	27.527
	26.469	27.527
10 <sup>+</sup>	23.823	25.942
	<u>29.117</u>	<u>29.117</u>
	30.705	30.706
	32.293	32.294
	33.881	32.294

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<sup>a</sup>In this table and in the following tables, the same value of  $\chi$  (and of  $\bar{\chi}$ ) was used for  $^{22}\text{Na}$  as for  $^{19}\text{F}$ .

TABLE III. Calculated  $B(E2)$  from the Ground State in  $^{22}\text{Na}$  with the  $-\chi Q \cdot Q$  Interaction.

$I = 1_1^+ \ T = 0 \rightarrow I = 2^+ \ T = 0$		
$E^*(I = 2^+, T = 0)$		$B(E2) \ (e^2 fm^4)$
1.591		34.89
1.598		4.31
3.199		0.00
8.995		0.00
10.054		0.00
$I = 1_1^+ \ T = 0 \rightarrow I = 3^+ \ T = 0$		
$E^*(I = 3^+, T = 0)$		$B(E2) \ (e^2 fm^4)$
1.570		6.15
1.586		48.74
3.183		0.00
5.306		0.00
5.320		0.00

TABLE IV. Calculated  $B(E2)$  Between Excited States in  $^{22}\text{Na}$  with the  $-\chi Q \cdot Q$  Interaction.

$I = 2^+ \ T = 0 \rightarrow I = 3^+ \ T = 0$		
$E^*(I = 2^+, T = 0)$	$E^*(I = 3^+, T = 0)$	$B(E2) \ (e^2 fm^4)$
1.591	1.570	0.00
1.591	1.586	13.73
1.591	3.183	3.18
1.598	1.570	13.5
1.598	1.586	0.00
1.598	3.183	26.57
3.199	1.570	1.44
3.199	1.586	0.20
3.199	3.183	0.00
$I = 2^+ \ T = 0 \rightarrow I = 4^+ \ T = 0$		
$E^*(I = 2^+, T = 0)$	$E^*(I = 4^+, T = 0)$	$B(E2) \ (e^2 fm^4)$
1.591	3.161	1.41
1.591	5.296	30.60
1.591	5.299	12.14
1.598	3.161	11.03
1.598	5.296	1.14
1.598	5.299	25.00
3.199	3.161	0.00
3.199	5.296	0.59
3.199	5.299	5.83
$I = 3^+ \ T = 0 \rightarrow I = 4^+ \ T = 0$		
$E^*(I = 3^+, T = 0)$	$E^*(I = 4^+, T = 0)$	$B(E2) \ (e^2 fm^4)$
1.570	3.161	40.10
1.570	5.296	0.15
1.570	5.299	2.68

1.586	3.161	5.03
1.586	5.296	4.37
1.586	5.299	1.73
3.183	3.161	0.00
3.183	5.296	2.48
3.183	5.299	24.61
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$I = 3^+ \ T = 0 \rightarrow I = 5^+ \ T = 0$		
$E^*(I = 3^+, T = 0)$	$E^*(I = 5^+, T = 0)$	$B(E2) \ (e^2 fm^4)$
1.570	5.278	20.31
1.570	5.289	0.36
1.570	7.945	0.00
1.586	5.278	8.74
1.586	5.289	36.00
1.586	7.945	0.00
3.183	5.278	4.53
3.183	5.289	0.20
3.183	7.945	25.09
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